Indian Maritime University

(A Central University, Govt of India)

May-June 2018 End Semester Examinations

B. Tech (Marine Engineering)

Semester-I

Mathematics I (UG11T3102)

Date: 05.07.2018 Max Marks:100 Marks

Time: 3 Hrs Pass Marks:50 Marks

Note : i) Use of approved type of scientific calculator is permitted.

ii) The symbols have their usual meanings.

Section –A (3× 10 = 30 marks)

Compulsory Question

Q 1

a) Find the entire length of the Cardioid $r = a(1 + cos\theta)$

b) What do you mean by change of order in case of double integration ? Explain with suitable example.

c) Find a unit vector normal to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$

d) Find the volume of a cone of radius R and height h , using integration.

e) Evaluate by Cauchy's integral formula $\oint \frac{z^2-z+1}{z-1}\, dz$ $, where\ C\ is\ a\ contour\ \ |z| = 1$

f) Prove that $\int_0^1 \frac{x^a-1}{logx}\, dx = \log\,(1 + a)\ \ ;\ a \geq 0$

g) Show that the radius of curvature for the rectangular hyperbola $xy = c^2$ is $\rho = \frac{(x^2+y^2)^{3/2}}{2c^2}$

h) Evaluate $\int_0^\infty \sqrt{y}\, e^{-\sqrt{y}}\, dy$

i) State Leibnitz's theorem and find n^{th} derivative of $y = \frac{x+2}{x+1} + \log\,(\frac{x+2}{x+1})$

j) Verify Cayley – Hamilton theorem for the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse.

Section B: Solve any **five** questions from the following

(14 × 5 = 70 marks)

Q 2

a) If $u = cosec^{-1}\sqrt{\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}}$, Show that

$x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \frac{tanu}{12}(\frac{13}{12} + \frac{tan^2u}{12})$ (7 marks)

b) Divide 24 into three parts such that the continued product of the first , square of the second and cube of the third may be maximum. Use Lagrange's method. (7 marks)

Q 3

a) If $x = sin\theta\ , y = sin2\theta$, Prove that $(1 - \mathrm{x}^2)y_{n+2} - (2\mathrm{n} + 1)\mathrm{x}y_{n+1} - (n^2 - 4)y_n = 0$

(7 marks)

b) Find the asymptotes of the curve

$y^3 - 2xy^2 - x^2y + 2x^3 + 3y^2 - 7xy + 2x^2 + 2y + 2x + 1 = 0$ (7 marks)

Q 4

a) Sketch the area of double integration and evaluate: (7 marks)

$\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} \log(x^2 + y^2)\, dxdy$

b) Using differentiation under integral sign , show that (7 marks)

$$\int_0^{\pi/2} \frac{\log(1 + asin^2x)}{sin^2x} dx = \pi[\sqrt{a+1} - 1]$$

Q 5

a) Show that $r^\alpha R$ is any irrotational vector for any value of α but is solenoidal if $\alpha + 3 = 0\ , where\ R = xi + yj + zk\ \ and\ r\ is\ the\ magnitude\ of\ R.$ (5 marks)

b) Find the values of $a\ and\ b$ such that the surface $ax^2 - byz = (a + 2)x$ and $4x^2y + z^3 = 4$ cut orthogonally at $(1\ , -1\ , 2\ )$ (5 marks)

c) If $uF = \nabla v$, *where* u *and* v are the scalar fields and F is a vector field , show that $F.curl\, F = 0$ (4 marks)

Q 6 (a and b carry 7 marks each)

a) Prove that the shortest distance between two points in a plane is a straight line.

a) Evaluate $\oint \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}$, where contour is the circle $|z| = 3$

Q 7

a) Investigate the values of λ *and* μ *so that the equations*

$2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$, have

i) No solution ii) a unique solution

iii) an infinite number of solutions. (7 marks)

b) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

(7 marks)

Q 8

a) Evaluate $\int \tan z \; dz$ over the contour $|z| = 2$ (7 marks)

b) Trace the curve $y^2(a - x) = x^2(a + x)$ (7 marks)

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